MATH-3 TEST 4 (Unit 4 – Polynomial, Rational and Conic Graphs

Fall 2023 SAMPLE

100 points   NAME:_____

Show all work on the test. On graphs, you are expected to use your knowledge about the function opposed to simply plotting points. No graphing calculators. LABEL ONE. POINT ON EACH GRAPH.

(1) Find the horizontal or slant asymptote (if any) for each of the following

(a) $f(x) = \dfrac{5x^3 + x^2 + x + 1}{7x^3 - 5x + 3}$ (b) $f(x) = \dfrac{2x^3 + x^2 + x + 1}{x^2 - 4}$ (c) $f(x) = \dfrac{x^2 - 4x + 2}{5x^3 + 1}$

(9 points)

$$y = \frac{5}{7}$$

$$\begin{array}{r} 2x+1 \\ x^2-4\overline{\smash{\big)}\,2x^3 + x^2 + x + 1} \\ -(2x^3 \quad - 8x) \\ \hline x^2 + 9x + 1 \\ x^2 \qquad -4 \\ \hline 9x + 5 \end{array}$$

$$\Rightarrow \text{slant } y = 2x+1$$

$$y = 0$$

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(2) Given the polynomial $P(x) = -2x^3(x+3)^2$ (12 points)

(a) what is the end behavior? $-2x^5$
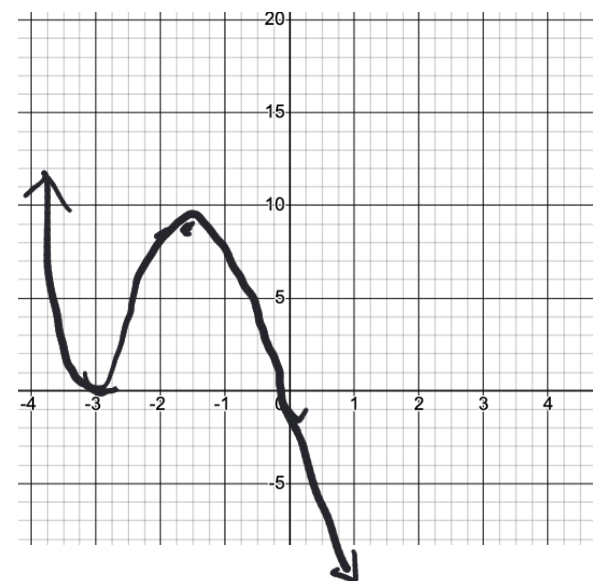
(b) find the y intercept 0

(c) find the x intercepts and discuss the behavior near them.
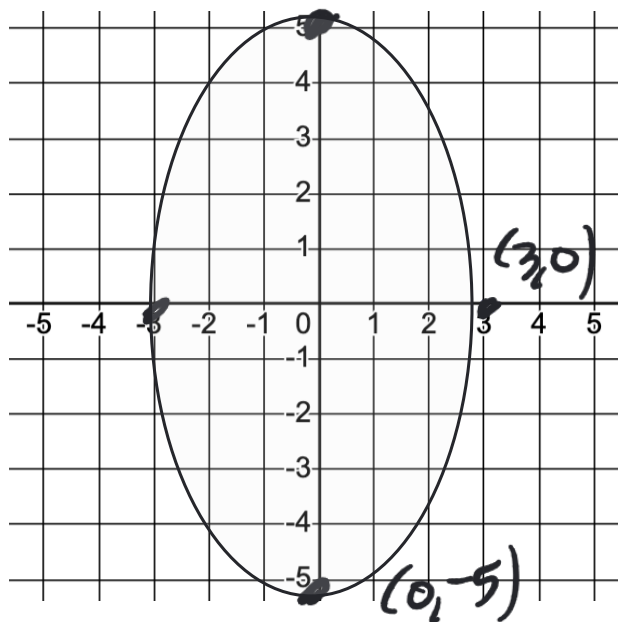
$x = 0$ cross

$x = -3$ bounce

(d) plot one additional point for accuracy and sketch the graph. SHOW ALL WORK

$P(-1) = 2 \cdot 4 = 8$

(3). Graph: $\dfrac{x^2}{9}+\dfrac{y^2}{25}=1$ (10 points)

$(3,0)$

$(0,-5)$

(4) Given the polynomial $f(x)=2x^4+15x^3+17x^2+3x-1$ (15 points)

$\pm\ 1, \frac{1}{2}$

(a) *Before* finding the zeros of f(x), list all POSSIBLE rational roots $\underline{\ \ \ \pm 1, \frac{1}{2}\ \ \ \ \ \ \ }$

(b) Find the zeros. Show all steps clearly including trying numbers that did not work.

Try
X=-1

$$\begin{array}{r|rrrrr} -1 & 2 & 15 & 17 & 3 & -1 \\ & & -2 & -13 & -4 & 1 \\ \hline & 2 & 13 & 4 & -1 & 0 \end{array}\ \text{WORKS}$$

X = -1 is a zero, X+1 is a factor

$f(x) = (x+1)(2x^3+13x^2+4x-1)$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 13 & 4 & -1 \\ & & -1 & -6 & 1 \\ \hline & 2 & 12 & -2 & 0 \end{array}\ \text{works}$$

so X = -½ is also a zero, x+½ a factor

$f(x) = (x+1)(x+½)(2x^2+12x-2)$

$\quad\quad = (x+1)(x+\frac{1}{2})2(x^2+6x-1)$

⇒ zeros   x+1=0   x+½=0 $x^2+6x-1=0$

X = -1   x = -½ $x = \dfrac{-6\pm\sqrt{37}}{2}$

(5) Match the function to its graph:

slant

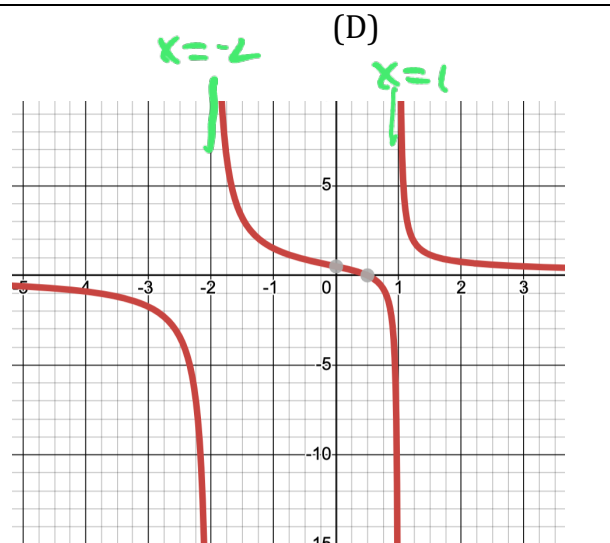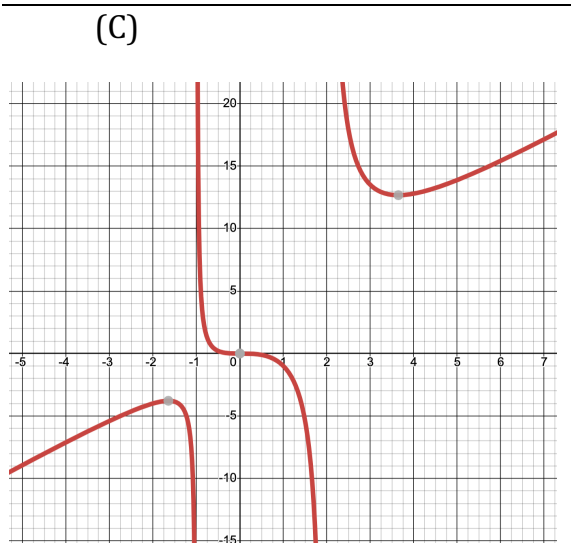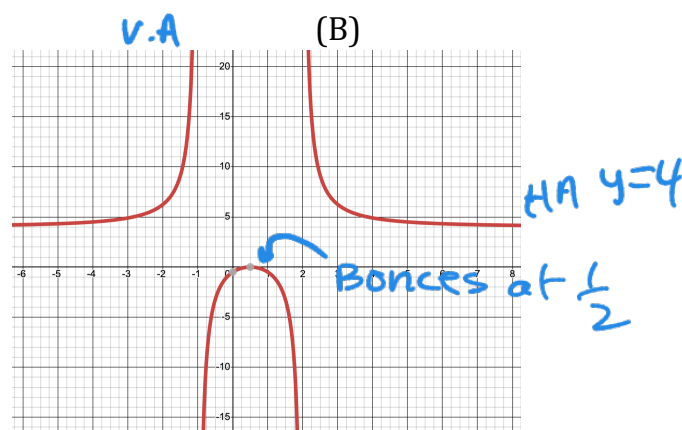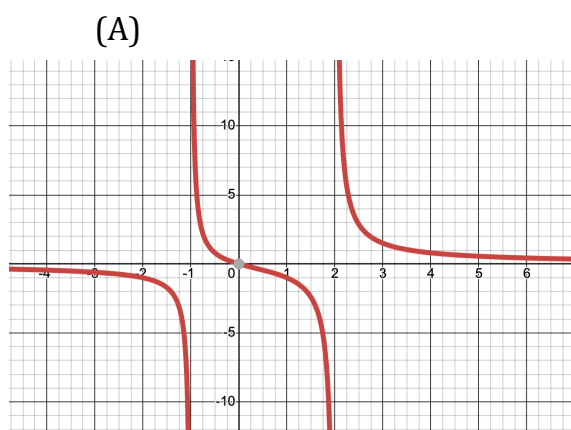a) $f(x) = \dfrac{x^3}{x^2 - x - 2}$ ___C___

H.A
x-axis

c) $f(x) = \dfrac{2x}{x^2 - x - 2}$ ___A___

a,b,c have same
(12 points) **V.A., x = -1, 2**

b) $f(x) = \dfrac{(2x-1)^2}{x^2 - x - 2}$ ___B___

d) $f(x) = \dfrac{2x-1}{x^2 + x - 2}$ ___D___

Note: Diff. V-A
(x+2)(x-1)
x = -2, 1

(A)

V.A   **(B)**

HA y=4

Bonces at ½

(C)

x=-2   x=1

(D)

(6) Graph $f(x) = \dfrac{x-1}{2x^2 - 3x - 2}$. Clearly show any asymptotes.

(15 points)

$x = -\frac{1}{2}$ $x = 2$
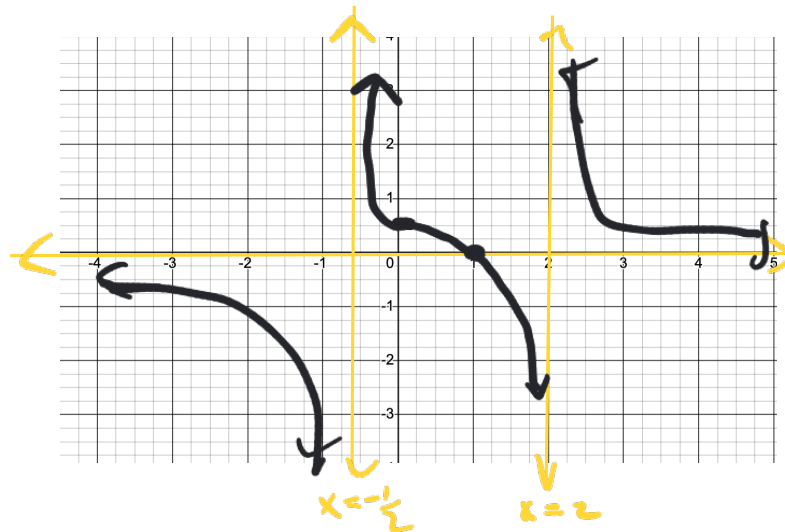
$f = \dfrac{x-1}{(2x+1)(x-2)}$

V.A x = -½, x = 2

H.A. y = 0

x-int: x = 1 cross

y-int ½

(7). Graph: $f(x) = 2x^3 - 7x^2 + 4x + 4$ (15 points)

y-int 4

end $\underset{\curvearrowleft}{x^1}$
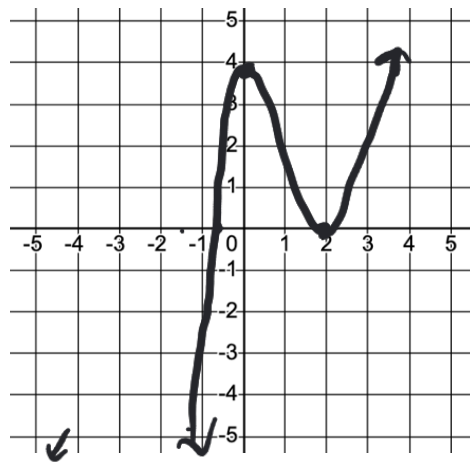
x-intercepts

$2x^3 - 7x^2 + 4x + 4 = 0$

$\left\{ 1, 2, 4, \frac{1}{2} \right\}$

$\begin{array}{c|cccc} -1 & 2 & -7 & 4 & 4 \\ & & -2 & 9 & -13 \\ \hline & 2 & -9 & 13 & \text{NO} \end{array}$

$\begin{array}{c|cccc} -\frac{1}{2} & 2 & -7 & 4 & 4 \\ & & -1 & 4 & -4 \\ \hline & 2 & -8 & 8 & 0 \end{array}$

$\begin{array}{c|cccc} \frac{1}{2} & 2 & -7 & 4 & 4 \\ & & 1 & 3 & 7\frac{1}{2} \\ \hline & 2 & -6 & 7 & \text{NO} \end{array}$

$K = -\frac{1}{2}$ is a zero

$X = -\frac{1}{2}$ is a zero so $X + \frac{1}{2}$ is a factor

$f(x) = (X + \frac{1}{2})(2x^2 - 8x + 8)$

$= (X + \frac{1}{2})(2)(X - 2)^2$

X-int, $-\frac{1}{2}$ cross
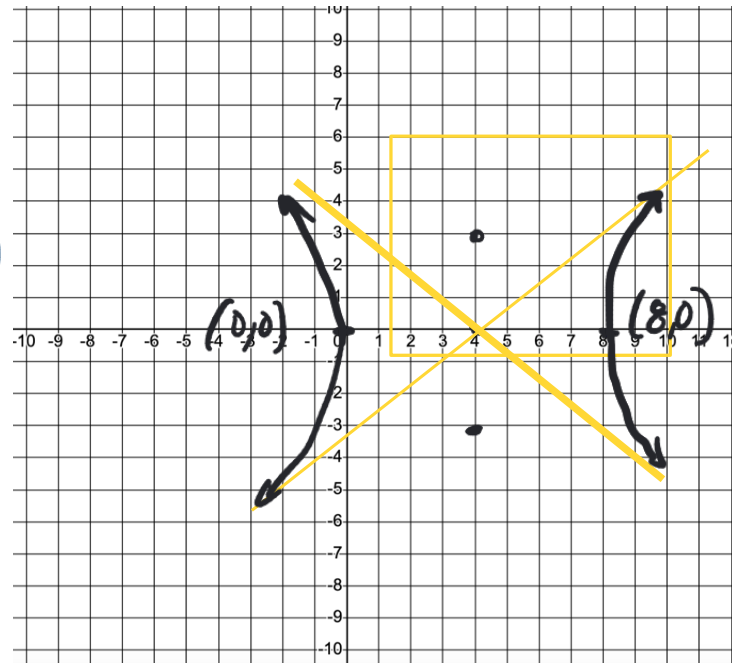2 bounce

(8) Graph. $9x^2 - 16y^2 - 72x = 0$ (12 points)

$9x^2 - 72x \quad -16y^2 = 0$

$9(x^2 - 8x + 16) - 16y^2 = 0 + 16(9)$

$9(X - 4)^2 - 16y^2 = 144$

$\dfrac{(X - 4)^2}{16} - \dfrac{y^2}{9} = 1$

(0,0)   (8,0)

Check a point

(0,0)? $9(0) - 16(0) - 72(0) = 0$ ✓